ESTIMATED PARAMETERS AS INDEPENDENT VARIABLES--
WITH AN APPLICATION TO THE COSTS OF
ELECTRIC GENERATING UNITS

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ABSTRACT

The cost of a piece of capital equipment, like an electric generating unit, is a function of a variety of unit-specific attributes. Some of these attributes can be observed directly without error (such as size), but others (such as the reliability or efficiency of the equipment), cannot be. However, estimates of the unobservable quality attributes can often be obtained from time series data on *ex post* performance, and these estimates can in turn be used as "data" on the unobservable attributes that appear as exogenous variables in a cost equation. We consider estimation of linear models in which observation-specific (firm, plant, household, individual) attributes appear as exogenous variables, but these attributes cannot be observed directly. Rather, we assume that estimates of the relevant observation-specific attributes, along with the associated covariance matrix, can be computed using data on variables (such as *ex post* performance) that do not appear directly in the primary model of interest. A maximum-likelihood technique for using such estimates as independent variables in cross-section regression analysis is derived. Our solution to the measurement error problem is interpretable as non-linear (Theil-Goldberger) mixed estimation. The method is applied to the estimation of a construction cost relationship for electric generating units.

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1. INTRODUCTION

This study was motivated by an estimation problem that arose in the analysis of the capital costs of electric generating facilities. The costs of constructing fossil-fueled generating facilities are generally thought to vary with the costs of inputs used to build the facility, along with an array of unit-specific attributes associated with each facility. Among the attributes commonly considered are unit size, the type of fuel burned, the presence of environmental control equipment, and the vintage of the facility. These attributes can normally be observed directly, along with the costs of construction, for each unit in the sample.

In addition to these attributes, it is also logical to assume that construction costs will vary with the "quality" of the facility. Two indices of quality are especially important in the case of electric generating facilities. These are the efficiency with which the unit transforms fuel into electricity (commonly measured by the unit's heat rate (HR)) and the reliability of the unit (commonly measured by the unit's equivalent availability (EAF)[2]). The more efficient is the unit (the lower the heat rate), the lower are the total costs of producing a unit of electricity, other things equal. The more reliable is the unit (the higher the equivalent availability), the larger the effective potential output of the facility given any design capacity and the lower are maintenance costs, again implying lower costs per unit of electricity produced.[3]

We can think of a generating unit construction cost function of the following form:

$$(1.1) \quad C = C(A_1, A_2, u)$$

where A_1 is a vector of attributes including unit size, input prices, fuel characteristics, etc; A_2 is a vector of quality attributes that the facility is designed to achieve, and u is a disturbance term. Other things equal, given a static technological opportunity set, we would expect that construction costs should vary directly with the design quality of the unit. That is, it should cost more to build a higher quality unit.

While the first set of attributes can generally be observed directly along with construction costs at the time a unit goes into service, the quality attributes generally cannot be observed directly *ex ante*. Rather, over time, we can only observe actual realizations for these quality attributes as the unit is utilized to produce electricity. Observed performance and design performance can be expected to be related to one another only with error, however. Other things equal, actual performance could be greater or less than design performance for at least two reasons. First, there is a natural stochastic relationship between engineering designs and the actual performance observed for a particular facility in much the same way as the observed failure rate for a piece of durable equipment, like a car or refrigerator, will vary across the units produced. Second, the actual utilization of the equipment may differ from its design specifications and this in turn will affect the observed performance. A unit that uses fuel different from that which it was designed to use or is used more or less intensively than had been planned is likely to yield actual performance that differs from design performance. To complicate matters further, actual performance will generally vary systematically over time. In particular, performance will generally deteriorate as a unit ages. Ideally, one would like some standardized "life-cycle" measure of the quality attributes that control for intra-unit variations in the variables that affect observed performance. But data on performance for individual units are generally not available for more than a few years and in many cases we cannot observe performance at the same times in the life cycles of all units. For example, we may have performance observations on a sample of units for the years 1969 to 1977, but these units may have been placed in operation in different years. For some units we may have observations on performance after they have been operating for five or six years, while for other units we may have observations on performance starting with their first year of operation.

Ideally, we would like to estimate "standardized" measures of the quality attributes of individual units that control for intra-unit variability in observed performance due to aging and variations in fuel characteristics and utilization and that are consistent across units. Given these estimates and their stochastic characteristics, we then want to incorporate them in the construction cost function (1.1) and to obtain consistent estimates of their effects along with those for the attributes that are directly observable. Thus, we can think of our problem as involving a two-stage estimation process. We first want to extract comparable estimates of design performance attributes and then incorporate these estimates in an appropriate way to obtain estimates of all the parameters in the cost function.

Efforts to incorporate quality attributes into the estimation of generating unit cost functions that appear in the literature have generally been quite <u>ad hoc</u>. The most common approach involves controlling for unit age by using performance data for only the first (or second, third, etc.) year of each unit's operation.[4] Observed performance for a single year or the average of two or three years is then assumed to measure the corresponding dimensions of unit quality without error. But this approach is hard to justify; even controlling for unit age, unit quality attributes interact with a host of other (observable and unobservable) variables to produce actual performance. Once this is acknowledged, it becomes difficult to justify using only one year's performance data or even a simple two- or three-year average for each unit and to ignore the fact that we can measure quality attributes only with some error from actual performance data. By using data on performance for several years as well as incorporating the stochastic properties of the estimated quality attributes, it is possible to obtain more precise estimates of the relationship between the cost of a capital facility and the quality of the facility.

Obtaining good estimates of the relationship between unit construction costs and performance as well as the relationship between observed performance and the variables that affect it is important for more than academic reasons. Many regulators have now observed that unit performance varies widely across generating units and have proposed various penalty/reward systems to encourage performance improvements.[5] It is therefore important to understand both how observed performance varies with operating characteristics such as age, plant utilization, and fuel characteristics and the relationship between estimates of design quality attributes and construction costs. To the extent that there is in fact a tradeoff between capital expenditures and unit quality implied by the data, regulatory concern about generating unit quality would suggest a regulatory strategy that focuses on design and construction decisions. It is possible, however, that the wide variations in observed performance do not reflect primarily decisions and expenditures made at the construction stage, but instead reflect "natural" random variations in actual performance from design performance, and/or unit-specific operating and maintenance decisions made after a unit has been placed in service that cannot be readily observed. If observed variations in performance reflect primarily "random" variations or utility-specific maintenance behavior, quite different regulatory strategies would be appropriate.[6]

Although we apply our estimating technique to data on electric generating units, it is of more general relevance to problems of estimating the relationship between the costs of durable goods and their quality as well as to a broader class of problems that arise in microeconomics. In general, whenever unobservable attributes of firms, industries, households, or plants can be estimated by regression techniques and then appear as independent variables in a model of interest, the techniques presented here can be usefully employed.

The paper proceeds in the following way. In the next section we specify a "two-stage" model that includes the basic construction cost equation that we are interested in estimating, along with an equation for estimating unit-specific performance attributes from time series observations on actual performance and operating characteristics, for a sample of generating units for which we also have construction cost information and information on the other unit attributes (A_1) that can be observed directly. Section 3 presents and discusses maximum-likelihood (ML) estimation of the parameters of this two- equation system. Section 4 shows that ML estimates are produced when a natural nonlinear extension of Theil-Goldberger [17, 18] mixed estimation is used to combine information from the two data sets involved. Section 5 shows how this approach can be used to estimate a construction cost equation for electric generating units that incorporates estimates of quality attributes. Section 6 provides a few concluding remarks.

2. THE MODEL

We conceptualize our problem as involving a two-stage estimation procedure. In the first stage, we are interested in extracting estimates of unit-specific quality attributes from observations on actual unit performance for multiple periods, along with observations on operating characteristics which affect observed performance in each period. In the second stage, we are interested in incorporating these estimates of unit-specific attributes into a contruction cost relationship.

Stage 1

A fixed effects model provides a natural way to incorporate performance data from multiple periods and to control for the effects of operating conditions on observed performance:

$$(2.1) \qquad Z = D\delta + W\gamma + \nu,$$

where Z is a Tx1 vector of performance observations on all units, D is a TxM matrix of unit-specific dummy variables, δ is a vector of unknown unit-specific quality attributes, W is a TxR matrix of observations on exogenous variables such as unit age and capacity utilization that vary over time for each unit and are likely to affect observed performance, γ is the corresponding coefficient vector, and ν is a Tx1 error vector. If the variables in W are properly defined, one can interpret the elements of δ as measures of quality or other unit-specific attributes (see the example in Section 5). As long as R is not large, one can exploit the special form of D to obtain an estimate of δ, d*, and of the corresponding MxM covariance matrix, V*, at relatively low cost even for large M.[7]

Stage 2

We are concerned with situations in which the unknown parameter vector δ enters a "second-stage" model, the parameters of which it is desired to estimate. If δ affects only the dependent variable(s) in such a model, use of the vector of "first-stage" estimates, d*, instead of the true parameter values generally induces heteroscedasticity. This can easily be handled by employing the estimated covariance matrix, V*. (See Saxonhouse [14, 15].) On the other hand, if δ is logically an <u>independent</u> variable, simple substitution of d* and use of ordinary least-squares (OLS) would produce biased and inconsistent estimates because of measurement error. The existing econometric literature provides no operational guidance in this situation.

In what follows we present and discuss a maximum likelihood (ML) solution to the measurement error problem for second-stage models with the following semi-linear structure:

(2.2) $\quad Y = X(\delta)\beta + \varepsilon,$

where Y is a Nx1 vector of unit-specific observations, X is an NxK matrix of independent variables involving the unknown vector δ, β is a Kx1 vector of parameters of interest, and ε is a disturbance with mean zero and scalar covariance matrix $\sigma^2 I$. In a simple model of this sort, corresponding to (1.1) above, Y might be observations on unit cost, δ might be an Nx1 vector of unit-specific intrinsic reliability values, and δ might enter X by simply being one column of that matrix. In any such model, use of d* instead of δ in OLS estimation of (2.2) would lead to biased and inconsistent estimates of β because d* measures δ with error.

3. MAXIMUM LIKELIHOOD ESTIMATION

The key to estimation in the presence of measurement error is generally the use of information about the distribution of that error.[8] Such information is provided here by the "first-stage" estimate, d*, of δ and its covariance matrix, V*, both of which would normally be obtained by estimation of (2.1) or some other model.

For the usual reasons, we treat V* as known. This is obviously more palatable the larger is T relative to M. Under normality, the log-likelihood function for the model described in Section 2 can be written as follows:

$$(3.1) \qquad L = [(M+N)\ln(2\pi)-\ln(|V^*|]/2 - (N/2)\ln(\sigma^2) - \varepsilon'\varepsilon/2\sigma^2 - (1/2)(\delta-d^*)'H^*(\delta-d^*),$$

where $H^* = V^{*-1}$, and ε is the residual vector from (2.2). One can interpret (3.1) in Bayesian terms as the logarithm of an expression proportional to the posterior density implied by uninformative priors on σ^2 and β and an informative prior on δ.[9]

The first-order conditions for maximization of L are the following:[10]

(3.2a) $\partial L/\partial\sigma^2 = (1/2\sigma^2)(\varepsilon'\varepsilon/\sigma^2 - N) = 0,$

(3.2b) $\partial L/\partial\beta = (1/\sigma^2)(X'Y - X'X\beta) = 0,$

(3.2c) $\partial L/\partial\delta = H^*(d^*-\delta) + (1/\sigma^2)B'\varepsilon = 0,$

where the NxM matrix B, which links the two components of the model, is the matrix of derivatives of $X(\delta)\beta$ with respect to δ:

(3.3) $B(\delta,\beta) = \{[\partial X(\delta)/\partial\delta_1]\beta,\ldots,[\partial X(\delta)/\partial\delta_m]\beta\}.$

If δ is an Nx1 vector that enters X simply by being its mth column, $B = \beta_m I$.

The information matrix corresponding to (3.2) has the following elements:

(3.4a) $E[(\partial L/\partial\sigma^2)^2] = N/2\sigma^4,$

(3.4b) $E[(\partial L/\partial\sigma^2)(\partial L/\partial\beta)] = E[(\partial L/\partial\sigma^2)(\partial L/\partial\delta)] = 0,$

(3.4c) $E[(\partial L/\partial\beta)(\partial L/\partial\beta)'] = (1/\sigma^2)(X'X),$

(3.4d) $E[(\partial L/\partial\delta)(\partial L/\partial\delta)'] = H^* + (1/\sigma^2)(B'B),$

(3.4e) $E[(\partial L/\partial\delta)(\partial L/\partial\beta)'] = (1/\sigma^2)(B'X),$

using the independence of ε and $(d^*-\delta)$ and the fact that d^* is unbiased. The matrix given by (3.4) is block-diagonal, with the element corresponding to σ^2 in a block by itself. Interest thus attaches to the other block of that matrix, given by

(3.5)
$$\Gamma = (1/\sigma^2)\left[\begin{array}{c|c} X'X & X'B \\ \hline B'X & B'B + \sigma^2 H^* \end{array}\right].$$

Following Rothenberg [13], the parameters of this model are locally identified if and only if Γ is non-singular. Assuming that δ is such that $X(\delta)'X(\delta)$ is non-singular, the formula for a partitioned inverse indicates that if Γ^{-1} exists it is given by

(3.6a)
$$\Gamma^{-1} = \sigma^2 \left[\begin{array}{c|c} (X'X)^{-1} + FGF' & -FG \\ \hline -GF' & G \end{array}\right], \text{ where}$$

(3.6b) $G = [B'MB + \sigma^2 H^*]^{-1}$,

(3.6c) $M = I - X(X'X)^{-1}X'$,

(3.6d) $F = (X'X)^{-1}X'B$.

Clearly if G exists, so does Γ^{-1}. Since M is symmetric and idempotent, the first term in brackets in (3.6b) can be written as $(MB)'(MB)$, making clear that it is positive semi-definite. Since H^* is positive definite, so is G^{-1}, establishing the non-singularity of Γ. The key to identification here is the information on δ provided by "first-stage" estimation of (2.1) or a related model.

Solutions to (3.2) can be computed using an efficient scoring approach as follows.[11] (Let us postpone a discussion of the properties of this estimator until the end of Section 4.) Let θ be the (K+M)x1 parameter vector $(\beta', \delta')'$, which will be estimated by $t = (b',d')'$.

(1) Set the starting value $\delta^{(0)}$ equal to d*. Apply least squares to (2.2) with δ=d* to obtain starting values of β, σ^2, and ε.

(2) At each step i in the iteration, substitute $\theta^{(i-1)}$ into (2.2) and use (3.2a) to obtain estimates of ε and σ^2. Using these, compute the gradient of L with respect to θ:

$$\nabla L^{(i-1)} = \begin{bmatrix} X'\varepsilon \\ \hline B'\varepsilon + \sigma^2 H^*(d^* - \delta) \end{bmatrix} . \tag{3.7}$$

Revise the current estimate of θ, $\theta^{(i-1)}$, as follows:

$$\theta^{(i)} = \theta^{(i-1)} + \lambda\Gamma^{-1}\nabla L^{(i-1)}, \tag{3.8}$$

where λ is chosen by a line search to maximize L($\theta^{(i)}$). As computation of the MxM matrix G is expensive for large M, it may be desirable not to update Γ^{-1} at each iteration. If M > N, the following formula simplifies computation of G:

$$G = \{V^* - V^*B'M[\sigma^2 I + MBV^*B'M]^{-1}MBV^*\}/\sigma^2. \tag{3.9}$$

(3) If convergence has not occurred, go back to the start of (2).

(4) If convergence has occurred, report t = $\theta^{(i)}$, use (2.2) and (3.2a) to obtain a final estimate of σ^2, revise Γ^{-1} and report it as the asymptotic covariance matrix of t.

The second term in the upper-left block of Γ^{-1} reflects the additional uncertainty about β due to measurement error in δ. Thus if $d = d^*$ and OLS and ML estimates of σ^2 are identical, the ML covariance matrix of b will exceed that implied by OLS estimation of (2.2). (That is, the difference between the two matrices will be positive semi-definite.) In most problems, however, d will not equal d^*, and the ML estimate of σ^2 will be less than the OLS estimate. (Intuitively, setting $d \neq d^*$ can increase L only by reducing $\varepsilon'\varepsilon$ and thus improving the second-stage fit.) Thus, the ML standard errors of the components of β may be greater or less than the corresponding OLS standard errors. On the other hand, the lower-right block of Γ^{-1}, the asymptotic covariance matrix of d, is always smaller than V^*. Estimation of (2.2) adds information about δ, which makes the ML estimate of that vector more precise than the OLS estimate from (2.1).

4. NONLINEAR MIXED ESTIMATION

The estimation problem here is to combine information from the two relevant samples, coresponding to (2.1) and (2.2), to estimate the unknown vector θ. Theil-Goldberger [17, 18] mixed estimation provides a general solution to such problems when the relations involved are linear. We must modify that technique to apply it here, because (2.2) is non-linear in θ. Let us therefore consider the natural extension of mixed estimation from generalized least squares (GLS) to nonlinear estimation via successive linearization.

At any stage in such estimation, linearization of (2.2) around the current estimate of θ, $\theta^{(i-1)}$, yields

$$(4.1) \qquad Y + B\delta^{(i-1)} = X\beta + B\delta + \varepsilon,$$

where $B = B(\delta^{(i-1)}, \beta^{(i-1)})$ and $X = X(\delta^{(i-1)})$. The information from first-stage estimation of (2.1) or a related model can be expressed as

$$(4.2) \qquad d^* = \delta + v,$$

where v has covariance matrix V*. In Bayesian terms, one can think of (4.2) and V* as prior information, which is to be combined with sample information via (4.1).

In order to obtain a new vector of mixed estimates, $\theta^{(i)}$, stack these two equations to obtain

$$(4.3) \qquad \left[\begin{array}{c} Y+B\delta^{(i-1)} \\ \hline d^* \end{array}\right] = \left[\begin{array}{c|c} X & B \\ \hline 0 & I \end{array}\right]\theta^{(i)} + \left[\begin{array}{c} e \\ \hline v \end{array}\right] .$$

The disturbance term in (4.3) has a block-diagonal covariance matrix, with upper-left block $\sigma^2 I$ and lower-right block V*. Application of GLS to (4.3), substitution for Y from (4.1), and comparison with (3.5) and (3.6) yield

$$(4.4) \qquad \theta^{(i)} = \theta^{(i-1)} + \Gamma^{-1}\nabla L^{(i-1)}.$$

That is, each iteration in the mixed estimation approach corresponds to an iteration in the efficient scoring approach of the previous section with λ set equal to one and Γ^{-1} revised at each step. Both approaches converge to the ML estimates if they converge. As a computational matter, the efficient scoring approach of Section 3 is likely to be superior, with convergence speeded by a line search over λ and costs reduced by not revising Γ at each iteration. Moreover, the coefficient covariance matrix corresponding to (4.3) is simply Γ^{-1}, the asymptotic covariance matrix associated with explicit ML estimation.

In most situations, the large-sample properties of ML estimators greatly enhance their attractiveness. Unfortunately, this is not one of those situations. The number of unknown elements of δ is likely to be proportional to N, the second-stage sample size, in most applications. This produces a classic "incidental parameters" problem. Asymptotic arguments in this model must involve letting T become large relative to M <u>and</u> letting N become large relative to K. Such arguments may not provide much reassurance to those working with small samples. The mixed estimation interpretation of the ML approach provides a rationale for the use of t and Γ^{-1} in small samples, a rationale that is strengthened by the Bayesian interpretation of (3.1).

5. AN APPLICATION: CONSTRUCTION COSTS OF ELECTRIC GENERATING UNITS

We are interested in estimating a generating unit construction cost function of the following form for coal-burning generating units:[12]

(5.1) AVCOST = f(SIZE, WAGE, BTU, TIME, EFF, REL), where

AVCOST = natural logarithm of unit average capital cost in 1965 dollars per kilowatt of capacity,

SIZE = natural logarithm of nameplate capacity in megawatts,

BTU = natural logarithm of unit-specific mean of BTU's per pound of coal burned,

WAGE = natural logarithm of regional construction wage in 1965,

TIME = year of first operation minus 1959,

EFF = design thermal efficiency of the unit (defined below),

REL = design reliability of the unit (defined below).

The first four right-hand variables correspond to A_1 in equation (1.1), and the last two to A_2 in that equation.

Most engineering calculations suggest that unit construction costs decline with unit size, other things equal.[13] The components of a generating facility are purchased in a national market, and we do not expect their costs for vary from unit to unit at a point in time. However, a large fraction of the total costs of a generating unit are associated with the actual costs of site preparation, component assembly, construction of structures and foundations, etc. on the site. This work is accomplished by construction workers hired in local and regional labor markets for a particular project. We therefore expect observed construction costs to vary with regional wage rates for construction workers.

Generating units are also typically designed to burn coal of a particular type. The existing literature ignores the likelihood that the characteristics of the coal burned might affect construction costs. Coal with a relatively high BTU content is likely to require lower capital expenditures to achieve a particular set of design performance standards. High BTU coal can be burned more efficiently with smaller surface areas, and higher BTU coal generally has fewer waste products that can gum up the boiler and furnace. There are also likely to be savings associated with coal handling and ash removal equipment. Other things equal, we expect that units burning high BTU coal will be less expensive than units burning low BTU coal. We use the variable BTU to measure the BTU content of the coal each unit was designed to burn. (We tried introducing other coal characteristics as well initially in our empirical work, but the BTU content of the coal captured essentially all of the explanatory power of the inter-unit variation in coal characteristics.)

We have observations on units built over a ten-year period and, although we deflate our data to reflect changes in input prices over time, real costs may increase or decrease over time due to technological change, increases or decreases in construction productivity, and changes in environmental restrictions, the responses to which we cannot measure directly. To capture this we include a trend variable to measure the year a unit was placed in operation.

If variations in observed thermal efficiency and reliability reflected conscious decisions made to "design in" higher or lower quality in the design and construction of generating units, we would expect to observe that appropriate measures of design efficiency and reliability derived from the actual performance data will help to explain variations in observed construction costs. In particular, higher thermal efficiency and reliability should be associated with higher construction costs. On the other hand, if observed variations in estimated design quality are random outcomes drawn from a sample of units with the same expected design quality, or reflect unmeasured operating and maintenance decisions made after the unit is placed in service, we should find that there is <u>no</u> relationship between estimated design quality and initial construction costs. In any event, we cannot observe design quality attributes directly, but must apply the procedure discussed in the previous sections to derive estimates of these variables from the observations on actual performance and operating characteristics that we do have.

We discuss the specific functional forms used to estimate (5.1) after we present the procedure for estimating the two quality attributes in which we are interested and after we discuss the data that we use.

To estimate (5.1) using the procedure presented above, we must first obtain estimates of each of the two design quality attributes that are hypothesized to enter the construction cost function, along with the corresponding covariance matrix. Corresponding to (2.1) above, we specify two equations of the following form:

(5.2a) $REL = D\delta_1 + W\gamma_1 + \nu_1$

(5.2b) $EFF = D\delta_2 + W\gamma_2 + \nu_2$

We follow Stewart [16] and specify the variables corresponding to the two quality attributes as:

REL = -ln(1 - equivalent availability),

EFF = -ln[(gross heat rate - 6000)/6000], so that the variables become infinite as quality approaches the highest levels that are theoretically possible.

The D's are matrices of unit-specific dummy variables that take on a value of one for a unit when the observations are associated with that unit and zero otherwise. The coefficients δ_1 and δ_2 are then the unit-specific design quality attributes we wish to estimate and that we use in the cost function to be estimated, (5.1). In addition to the dummy variables, we enter several exogenous variables (W) that should affect intra-unit variations in actual operating performance over time. These variables are constructed to yield a consistent set of "standardized" estimates of unit-specific performance attributes that are readily interpreted. The variables included in w are the following:

∂CAPU = deviation of output factor [= 100 x generation/(capacity x hours in service)] from the sample mean for <u>all</u> units, a measure of (relative) capacity utilization.

AGE = unit age (calendar year—year of first operation) minus three.

∂BTU (∂MOIST, ∂ASH, ∂SULPH) = deviation of BTU's per pound (percentage of moisture, ash, sulphur) of coal burned from the sample mean for <u>this</u> unit.

Given this specifiation of the variables, the estimated coefficients of the unit-specific dummy variables in (5.2) are therefore estimates of the unit-specific design thermal efficiency and reliability for units that are three years old, at the sample mean output factor, burning coal with (unit-specific) average characteristics.

We expect to find that both EFF and REL will decline with age. We enter AGE quadratically to allow for the possibility that performance increases in the earliest operating periods as the units are "broken in." We also expect that observed performance in both dimensions will be higher if units are kept fully loaded (while available) rather than cycled up and down. With cycling, there is always a sacrifice in thermal efficiency, and cycling places more wear and tear on the unit, leading to a higher forced outage rate and lower availability. We do not have any priors on exactly how intra-unit variations in coal characteristics should affect observed performance. Some of the intra-unit variation in coal characteristics may reflect responses to environmental requirements after 1970, but some of it no doubt reflects responses to changing coal prices as well. Different coal characteristics are likely to affect performance in different ways.

The Data

Our choice of sample was governed by several considerations. First, data on unit operating performance is fairly hard to come by. We made use of a data base on the operating performance of coal-burning units discussed in Corio [4,5]. That data base contained operating performance data for about 150 units for the years 1969 through 1977. Second, we were interested in examining units for which we had several years of observations on operating performance. Corio had observations on some units built as recently as 1974, but relatively few observations on performance for these later units. Third, for purposes of this example, we wanted to restrict ourselves to "mature" technologies to avoid having to deal with potential learning effects and the dynamics of moving to a new technology.[14] Fourth, we wanted to avoid dealing with the changes in unit costs and in design operating performance associated with more stringent environmental regulations that began to affect the design of units during the 1970s. Finally, we had to restrict ourselves to units for which we also had construction cost data and observations on the other characteristics of the units. We drew these data from the coal generating unit data base discussed in Joskow and Rose [10].

These considerations led us to focus on sub-critical generating units placed in service between 1960 and 1969. We have observations on 71 such units,[15] with capacities ranging from 218 MW to 709 MW. Several of these units were "twins" in the sense that they were identical units built at the same plant either in the same year or adjacent years. Differences in reported construction costs between these units reflect accounting treatment that loads common costs onto the first unit placed in operation.[16] If the units were placed in operation in the same year, the reported construction costs were identical for the two units. For the purposes of estimating design performance attributes, it seemed sensible to assume that these units had identical attributes, so we first estimated the "first-stage" model constraining the estimated values of the unit-specific dummy variables to be the same for "twins". We then performed F-tests to see if we could reject the null hypothesis of identical performance attributes for twins. In all cases the F-statistics were less than one and thus the null hypothesis could not be rejected at conventional levels of significance. This left us with 58 observations on individual and consolidated "twin" units for the sample period. The construction cost data are expressed in constant 1965 dollars excluding interest during construction. Costs for units identified as "twins were averaged. The procedure for deflating the observed nominal construction cost data is described in Joskow and Rose [10] and is similar to the procedure used by Zimmerman [21] for nuclear plants.[17]

Limiting our analysis to subcritical units built during the 1960s is not without some costs. These units consist primarily of units with design steam pressures of 2400 psi and steam temperatures of 1000°F plus a few units with design steam pressures of 1800 psi and temperatures of 1000°F. The maximum theoretical variation in design thermal efficiency, other things equal, is probably less than 5% for units with these design specifications.[18]

First-Stage Results

Table 1 presents the OLS estimates of the parameters in γ_1 and γ_2 for two specifications of equations (5.2). These estimates show quite clearly that there is a statistically significant deterioration in unit performance as units age and that there is at most a very short "break in" period. Reliability peaks after a year or less of operation (recall that AGE is actual age minus three) and then declines, while efficiency declines from the start of operations. After ten years of operation, equivalent availability declines between 7% and 9%, while the heat rate rises (thermal efficiency falls) by about 4%. A unit that operates continuously when it is available has about a 20% higher availability and a 3% greater thermal efficiency than a unit that is cycled up and down and has an output factor of only 50%. (These comparisons are made about the point of sample means.) These results suggest that, at the very least, regulators interested in basing penalties and rewards for unit efficiency on comparisons with other units must take account of unit ages and operating characteristics.

Intra-unit variation in coal characteristics appears to have little effect on observed generating unit performance. (Alternative specifications using, for instance, the squares of deviations from unit mean coal characteristics do not improve the results.) The coal characteristic variables are never significant individually or as a group. We found this to be surprising, since discussions with knowledgeable people in the industry emphasized the importance of coal quality for generating unit performance. The observed intra-unit variations in coal quality may simply be too small to affect performance noticeably.

(Inter-unit variations in coal qualiy are much larger than intra-unit variations. To the extent that inter-unit variations have more important effects on performance, these effects are embedded in the unit-specific quality attributes estimated for each unit. Because coal characteristics are taken into account at the design stage, we felt that the appropriate way to introduce inter-unit variations in average coal characteristics was directly into the (second-stage) cost function, as discussed above. This allows us to identify the effects of increases or decreases in unit-specific performance attributes given the characteristics of the coal that the unit was designed to burn.)

These results also appear to be inconsistent with the view that changes in coal characteristics required by State Implementation Plans governing units placed in operation during the 1960s (to achieve reductions in sulphur and particulate emissions in the early 1970s as required by the Clean Air Act Amendments of 1970) are responsible for the deterioration in generating unit performance during the 1969-1977 time period. To the extent that there are any effects associated with intra-unit variations in coal quality during this period, they appear to be relatively small. Moreover, the effects of any reductions in ash and sulphur content necessitated by these regulations move in opposite directions. This issue is worthy of further investigation with an expanded data base, including specific information on constraints imposed by the State Plans on each unit. Perhaps with more data we will be able to estimate the effects of intra-unit variations in coal characteristics with more precision.

In addition to the coefficients reported in Table I, coefficients of 58 unit-specific dummy variables were estimated in both EFF and REL equations. (These correspond to δ_1 and δ_2 in equations (5.2).) F-tests decisively rejected the null hypotheses that unit qualities were identical. The estimated unit-specific values of REL from the equation using coal characteristics correspond to equivalent availabilities ranging from .60 to .97, with a mean of .84, and an approximate standard deviation of .09. Similarly, the estimates of EFF from the equation with coal characteristics imply unit-specific heat rates ranging from 7,700 to 10,800, with a mean of 9,000 and an approximate standard deviation of about 500. Estimates from the equations without coal characteristics were nearly identical. The variation in these estimates is quite large, substantially larger than we would expect merely from variations in ex ante performance attributes. It appears that even after standardizing the performance data for time-varying factors that affect observed performance, the estimated unit-specific quality attributes still have a large random component.

Finally, it is worth emphasizing that the regressions in Table I explain less than 60% of the variance of units' observed performance, even with the inclusion of unit-specific dummy variables. The assumption that any one year's performance measures unit-specific quality without error is clearly untenable.

In order to obtain d* and V* for use in our second-stage capital cost equation, the REL and EFF equations had to be pooled and estimated jointly. This is complicated by the presence of more observations on REL than on EFF. The GLS/ML procedure employed to compute d* and V* is described in the Appendix.

Second-Stage Results

We now turn to the estimation of the "second stage" construction cost function. In what follows we focus on estimates using d* and V* produced by the "first stage" equations containing coal characteristics. The results using the equations without the coal characteristics are virtually identical. We report results for three different estimates of two specifications of the construction cost equation (5.1) in Table III. The first ("Linear") specification is linear in the variables listed below equation (5.1), except for the time effects, which are entered linearly as TIME and $TIME^2$. (CONST is the intercept term.) The more complex ("Interactive") specification allows the effects of changing reliability or efficiency to depend on the level of the other variable and on the quality of coal burned.[19]

For each specification, we first give OLS estimates obtained by using unit-specific averages of observed EFF and REL as the two sub-vectors of δ, rather than the values estimated in the first stage. (We cannot examine the most simplistic estimation approach, which would simply use each unit's first or second year's operating performance, since we do not have observations on performance for any single "age" for all units.) The second set of estimates, reported in the columns headed "OLS-d*" are OLS estimates obtained by setting the first-stage estimates, d*, equal to δ. Finally, the columns headed "ML" provide the corresponding maximum-likelihood estimates.

Turning to the results reported in Table III, it is clear that several of the characteristics of the construction cost relationship are not particularly sensitive to specification or estimating technique.[20] First, construction costs exhibit statistically significant and empirically important scale economies. Doubling capacity is estimated to reduce average construction cost by 10 to 12%. Furthermore, there is no evidence that the average cost function reaches a minimum within the range of the sample observations. (When we enter $SIZE^2$ in the equation its coefficient is sometimes positive, but never is statistically significant.) Second, units that are located in areas where they can readily make use of high-BTU coal incur significantly lower capital costs, other things equal, than units making use of low-BTU coal. Increasing the BTU content of the coal from the sample minimum to the sample maximum is expected to lower average construction cost by about 4% given mean values of EFF and REL.[21] The relationship between coal quality and construction cost does not appear to have been considered in the previous literature, yet this relationship appears to be of some importance. Third, higher local construction wage rates raise construction costs substantially.

Costs are between 11% and 15% lower in areas with the lowest wages compared to areas with the highest wages.

Fourth, the estimated quadratic time trend is both robust and important. Dropping $TIME^2$ from essentially any specification causes the coefficient of TIME to lose significance. The estimates in Table III imply a decline in <u>real</u> costs (recall that the cost data have been corrected for input price changes) of over 30% between 1960 and 1966. This is consistent with the historical record of cost-reducing technological change in the construction of generating facilities. However, real costs stop declining around 1966 and then begin to increase. Between 1966 and 1969 average construction costs increase by about 14%.[22] It is not clear what caused the observed increases in real costs. The units in the sample were all built prior to the era of stringent environmental constraints, so that we cannot attribute the cost increases to environmental regulation. The most likely hypothesis is that these cost increases reflect declines in on-site construction productivity that are perhaps attributable to changes in contract construction practices in the late 1960s.

Finally, the quality attributes, EFF and REL, do not appear to affect construction costs significantly. In no equation do they contribute significantly to the explanatory power of the regression. In the linear specification the estimated coefficients are extremely imprecise and always have the wrong signs. In the interactive specification, the derivative of average cost with respect to REL is positive at the sample means, but the effect of increases of EFF on costs remains negative.[23]

These results suggest that there is very little actual variation in design quality among generating units in our sample that would be revealed in the data as an observable tradeoff between cost and quality. This is consistent with the finding of Joskow and Rose [10] that there is little, if any, difference in construction costs between subcritical units with different design steam pressures. The wide variation in the estimates of unit-specific intrinsic quality appears to reflect instead some combination of inherent randomness in actual unit performance given specific design parameters (some units turn out to be lemons and others to be superior performers despite identical design characteristics and construction costs) plus unmeasured variations in utilization and maintenance behavior of utilities once the units are placed in service. It is also possible that there are systematic utility and/or Architect-Engineer (AE) specific variations in *ex post* performance, given the level of construction costs and design performance levels, that we cannot capture here given the limited data set that we have available and are therefore simply reflected in the variation in estimated unit-specific quality levels. In either case, this suggests to us that regulators concerned about poor unit performance would be best advised to focus their attention on the way utilities use and maintain their units rather than worrying about tradeoffs between cost and quality at the design stage. In addition, further analysis of systematic variation in performance associated with specific utilities or AE's would also be desirable.

To the extent that there are any observable relationships between construction cost and unit quality attributes, the method of estimation that we develop here appears to be superior to the simple ad hoc approaches that have been used in the past. While standard F-tests using both pairs of OLS estimates fail to reject the null hypothesis that the last three coefficients in the interactive specification are zero, a large-sample likelihood ratio test rejects that hypothesis at the 5% level ($\chi^2(3) = 8.03$) using ML estimates. Within each specification, second-stage fit improves as one moves from left to right in Table II. Some improvement from "OLS-d*" to "ML" is essentially guaranteed; ML estimation moves d away from d* in order to improve second-stage fit. (From (2.1), increases in the precision of first-stage estimation increase the penalty for doing this and thus keep d closer to d*.) The improvement from "OLS-Means" to "OLS-d*" presumably reflects the superiority of fixed-effects estimation to simple averaging.

Estimates of ß are generally more sensitive to choice of method for the interactive specification than for the linear specification, presumably because the latter is more sensitive to changes in REL or EFF. Similarly, the ML estimate of δ differs more from d* in the interactive than in the linear specification.[24] In both specifications the differences between the "OLS-d*" and "ML" estimates of ß seem roughly as important as the differences between the "OLS-Means" and "OLS-d*" estimates. The ML estimates show scale economies to be less important than the OLS-d* estimates, particularly for the interactive specification. Coefficients of terms involving EFF and REL are most sensitive to choice of estimation method, as one might expect. We derive some encouragement from the fact that the coefficient of REL at the point of sample means in the interactive specification moves in the expected direction as one goes from least- to most-preferred estimation technique. But even using the ML estimates, the estimated derivative of AVCOST with respect to REL is negative over much of the sample range. Finally, consistent with the discussion at the end of Section 3, most ML standard errors are larger than the corresponding OLS-d* estimates, but some are smaller.

6. CONCLUDING REMARKS

We have presented a maximum-likelihood technique for using estimated parameters as independent variables in linear regression. Our development of this technique was motivated by the unobservability of the quality (efficiency and reliability) of electric generating units. The application of our approach to data on the construction cost of coal-fired generating units produced some positive results and exhibited some of the properties of maximum-likelihood estimators in this context, though it left us with a number of empirical problems for future research.

The results suggest several areas for future research. First, it would be useful to expand the data set to include a larger number of units, including supercritical units, as well as more observations on performance for each unit in the sample. With a larger number of units, especially with the inclusion of supercritical units, the variation in engineering design efficiencies would be expanded. It would also be possible to test for systematic utility- and AE-specific performance differences. Additional observations on unit performance would allow us to obtain more precise estimates of unit-specific performance attributes as well. Expanding the sample in these dimensions is a major task, but the results appear to us to be of sufficient interest to at least make an effort to do so.

Though we have discussed the use of estimated parameters as independent variables in the context of cost function estimation, applications of the technique developed here are not restricted to that context. For instance, a number of studies in industrial economics have used intra-industry regressions to estimate unobservable industry characteristics such as the relation between market share and profitability.[25] Such intra-indusry regressions correspond to least-squares estimation of (2.1) here, except that the coefficients of non-dummy variables may be of primary interest. The general approach developed here is well-suited to models in which estimable industry-specific parameters appear as or affect independent variables in models of the form of (2.2). In general, whenever unobservable attributes of firms, industries, households, plants, or other units can be estimated by regression and appear as independent variables in a model of interest, the techniques presented here can be usefully employed.

APPENDIX

The first-stage model employed in the application discussed in Section 5 can be written as follows:

$$
\text{(A.1)} \quad \begin{bmatrix} Z^1 \\ \hline Z^2 \\ \hline Z^{2E} \end{bmatrix} = \left[\begin{array}{c|c|c|c} D & W & 0 & 0 \\ \hline 0 & 0 & D & W \\ \hline 0 & 0 & D^E & W^E \end{array}\right] \begin{bmatrix} \pi^1 \\ \hline \pi^2 \end{bmatrix} + \nu,
$$

where Z^1 is a Tx1 vector of observations on EFF, Z^2 is a Tx1 vector of matched (in terms of units and years) observations on REL, Z^{2E} is an Ex1 vector of "extra" observations on REL, D and D^E are matrices of unit dummy variables, W and W^E are matrices of other variables, $\pi^1 = (\delta^{1\prime}, \gamma^{1\prime})'$, $\pi^2 = (\delta^{2\prime}, \gamma^{2\prime})'$, and ν is a (2T+E)x1 disturbance vector with covariance matrix

$$
\text{(A.2)} \quad S = \left[\begin{array}{c|c|c} \sigma_{11} I_T & \sigma_{12} I_T & 0 \\ \hline \sigma_{21} I_T & \sigma_{22} I_T & 0 \\ \hline 0 & 0 & \sigma_{22} I_E \end{array}\right]
$$

Treating Σ as known and maximizing the likelihood function coresponding to (A.1), one can show after a good deal of algebra that the ML estimate of π^2, p^2, is obtained by simply applying OLS to the REL equation, using all T+E observations. Call this estimate $p^{2(T+E)}$, and let $p^{2(T)}$ be the estimate obtained by applying OLS only to the T observations for which EFF is also available. Letting $p^{1(T)}$ be the corresponding estimate of the EFF coefficients, the ML estimate of π^1 can be shown to be given by

$$
\text{(A.3)} \quad p^1 = p^{1(T)} + \rho\sqrt{\sigma_{11}/\sigma_{22}}\,[p^{2(T+E)} - p^{2(T)}],
$$

where $\rho = \sigma_{12}/\sqrt{\sigma_{11}\sigma_{22}}$ is the correlation between the two residuals. If $\rho = 0$ or if $E = 0$, OLS and ML estimates are identical, since both equations have the same independent variables. The first N elements of p^1 and the first N elements of p^2 make up the vector d^* used in second-stage estimation. Because estimates of ρ were near zero in the application discussed in the text, differences between p^1 and $p^{1(T)}$ were quite small, and only the OLS estimates of γ are presented in Section 5.

After a bit more algebra, one can verify that the covariance and precision matrices corresponding to d^* are as follows:

$$\text{(A.4a)} \quad V^* = \left[\begin{array}{c|c} \sigma_{11}[(1-\rho^2)f+\rho^2 f^*] & \sigma_{12}f^* \\ \hline \sigma_{21}f^* & \sigma_{22}f^* \end{array}\right],$$

$$\text{(A4.b)} \quad H = \left[\begin{array}{c|c} \sigma^{11}f^{-1} & \sigma^{12}f^{-1} \\ \hline \sigma^{21}f^{-1} & \sigma^{22}[(1-\rho^2)f^{*-1}+\rho^2 f^{-1}] \end{array}\right],$$

where the σ^{ij} are the elements of the inverse of the 2x2 matrix with typical element σ_{ij}, the matrices f and f^{-1} are defined as follows:

(A.5a) $f = Q + QD'W[W'(I-DQD')W]^{-1}W'DQ$,

(A.5b) $f^{-1} = D'[I-W(W'W)^{-1}W']D$,

(A.5c) $Q = (D'D)^{-1}$,

and f^* and f^{*-1} are defined by (A.5) using $D^* = [D : D^E]$ in place of D and $W^* = [W : W^E]$ in place of W.

REFERENCES

[1] Bushe, Dennis M.: "An Empirical Analysis of Production and Technology Using Heterogeneous Capital: Thermal Electric Power Generation," Ph.D. dissertation, New York University, 1982.

[2] Caves, Richard E., and Pugel, Thomas E.: Intraindustry Differences in Conduct and Performance: Viable Strategies in U.S. Manufacturing Industries. New York: New York University Graduate School of Business, Solomon Brothers Center for the Study of Financial Institutions, 1980.

[3] Comanor, William S., and Wilson, Thomas A.: Advertising and Market Power. Cambridge: Harvard University Press, 1974.

[4] Corio, Marie R.: "Heat Rates and Availability: The Use of Econometric Analysis in Comparing Electric Generating Unit Performance," mimeographed, National Economic Research Associates, Inc., Summer 1981.

[5] ____________: "Why is the Performance of Electric Generating Units Declining?" Public Utilities Fortnightly, 29 April 1982, 3-8.

[6] Cowing, Thomas G., and Smith, V. Kerry: "The Estimation of a Production Technology: A Survey of Econometric Analyses of Steam-Electric Generation," Land Economics, 54 (1978), 156-186.

[7] Daskin, Alan J.: "Essays on Firm Diversification and Market Concentration," Ph.D. dissertation, M.I.T., 1984.

[8] Dhrymes, Phoebus, J.: Introductory Econometrics. New York: Springer-Verlag, 1978.

[9] Gordon, Robert J.: "The Productivity Slowdown in the Steam-Electric Generating Industry," mimeographed, Northwestern University, February 1983.

[10] Joskow, Paul L., and Rose, Nancy L.: "The Effects of Technological Change, Experience, and Environmental Regulation on the Construction Costs of Coal-Burning Generating Units," mimeographed, MIT, March 1984.

[11] Perl, Lewis: "The Current Economics of Electric Generation from Coal in the U.S. and Western Europe," mimeographed, National Economic Research Associates, Inc., October 1982.

[12] Rao, C.R.: Linear Statistical Inference and Its Applications, 2nd ed., New York: John Wiley, 1973.

[13] Rothenberg, Thomas J.: "Identification in Parametric Models," Econometrica, 39 (1971), 577-592.

[14] Saxonhouse, Gary R.: "Estimated Parameters as Dependent Variables," American Economic Review, 66 (1976), 1978-1982.

[15] __________: "Regressions from Samples Having Different Characteristics," Review of Economics and Statistics, 59 (1977), 234-237.

[16] Stewart, John F.: "Plant Size, Plant Factor, and the Shape of the Average Cost Function in Electric Power Generation: A Non-homogeneous Capital Approach," Bell Journal of Economics, 10 (1979), 549-565.

[17] Theil, H.: "On the Use of Incomplete Prior Information in Regression Analysis," Journal of the American Statistical Association, 58 (1963), 401-414.

[18] ________, and A.S. Goldberger: "On Pure and Mixed Statistical Estimation in Economics," International Economic Review, 2 (1961), 65-78.

[19] Wills, Hugh R.: "Estimation of a Vintage Capital Model for Electricity Generating," Review of Economic Studies, 45 (1978), 495-510.

[20] Zellner, Arnold: An Introduction to Bayesian Inference in Econometrics. New York: John Wiley, 1971.

[21] Zimmerman, Martin B.: "Learning Effects and the Commercialization of New Energy Technologies: The Case of Nuclear Power," Bell Journal of Economics, 13 (1982), 297-310.

FOOTNOTES

1. The authors are indebted to the U.S. Department of Energy for financial support, to Marie Corio and Lewis Perl for supplying data, to Ademola Aderibibge, Ben Golub, and, especially, Nancy Rose for superb research assistance, and to Jerry Hausman for invaluable guidance. The usual disclaimer applies.

2. Equivalent availability, as defined by the National Electric Reliability Council, the source of our reliability data, is the fraction of the period that a unit was available to generate power, adjusting for partial outages that reduced effective capacity. The (gross) heat rate is equal to the number of BTU's consumed per kilowatt-hour of generation.

3. Poor reliability is generally associated with higher maintenance costs. Unit-specific maintenance expenditures are not reported anywhere, as far as we know. Poor availability also increases the costs of generating electricity in both the short run and the long run since it necessitates using more costly backup generating capacity or increased wholesale power purchases and reduces the effective dependable capacity of the generating system, requiring additional investment in capacity to maintain system reliability targets.

4. Cowing and Smith [6] provide an excellent survey of much of the relevant literature. Interesting recent studies of steam-electric generation costs include Bushe [1], Gordon [9], Perl [11], Stewart [16], and Wills [19].

5. See, for example, "Rate Incentive Provisions: A Framework for Analysis and a Survey of Activities," National Regulatory Research Institute, Columbus, Ohio, 1981.

6. For example, by scrutinizing utility maintenance practices more carefully.

7. Let $Q = (D'D)^{-1}$, a diagonal matrix, and let $W^o = [I-DQD']W$ and $Z^o = [I-DQD']Z$. The last two matrices are formed by subtracting unit-specific means of each variable. The OLS estimate of γ, $c^* = (W^{o\prime}W^o)^{-1}W^{o\prime}Z^o$, is obtained by regressing Z^o on W^o. The standard error of that regression, corrected for the estimation of $(M+R)$ instead of R parameters, provides the usual estimate of $\sigma(\nu)$. Let $\underline{Z} = QD'Z$ and $\underline{W} = QD'W$; these are $M\times 1$ and $M\times R$ matrices of unit-specific means. Then $d^* = \underline{Z} - \underline{W}c^*$, with covariance matrix $V^* = \sigma^2(\nu)[Q + \underline{W}(W^{o\prime}W^o)^{-1}\underline{W}']$. It is worth noting that most discussions of fixed effects estimation assume that γ contains the parameters of interest, with δ being essentially a vector of nuisance parameters. Here, however, δ matters because it enters the equation of primary interest, (2.2).

8. Dhrymes [8, Ch. 5] and Zellner [20, Ch. 5] provide good discussions of estimation in the presence of measurement error. As the expression for V^* in footnote 7 makes clear, our problem can be reduced to the standard one of i.i.d. errors if and only if we have the same number of observations on each unit and $D'W = 0$.

9. See, for instance, Zellner, [20, pp. 70-2]. Under the usual uninformative prior on σ, the coefficient of $\ln(\sigma^2)$ in (3.1) would be (N+1)/2 rather than N/2.

10. Since (2.1) and (2.2) are two parts of a single model, linked by the common parameter vector δ, application of OLS to (2.1) will not yield ML estimates of γ. If (3.1) is expanded to include γ, it can be shown that ML estimates of that vector are as follows. Let c* and d* be the OLS estimates of γ and δ, respectively, and let c and d be the corresponding ML estimates. Finally, let H_{cc} be the inverse of the RxR diagonal block of the coefficient covariance matrix from (2.1) corresponding to γ, and let V_{cd} be the RxM off-diagonal block of that matrix. Treating these two matrices as known, one can establish $c = c^* + V_{cd}H_{cc}(d-d^*)$. After equations (3.2) are solved for d, this relation can be used to compute c. A bit more manipulation along the lines of (3.4) yields the corresponding asymptotic covariance matrix, which includes Γ^{-1} from (3.6).

11. See, for instance, Rao [12, Sect. 5g]. The estimates reported in Section 5 were computed using a program, RANDHRS, written in FORTRAN to run on the Prime 850 at MIT. (Listings are available on request.) RANDHRS allows δ to be either Nx1 or 2Nx1 and allows X to be a linear or quadratic function of δ and a set of variables observed without error. RANDHRS allows the user to use the method of steepest descent or to mix that method with efficient scoring. This permits at least some search for multiple local maxima.

12. Generating units, each of which consists of a generator, a turbine, a boiler, and the associated fuel-handling equipment, are the natural units of analysis in this industry. Generating plants typically house multiple units, often of very different scales and vintages.

13. The relevant literature is discussed in Joskow and Rose [10].

14. These considerations are discussed in Joskow and Rose [10].

15. Units built since 1960 fall into two broad categories—subcritical and supercritical—depending on their design steam pressures. Supercritical technology was first introduced commercially in the 1960s, while even the most efficient subcritical designs were already being introduced by the mid-1950s. See Joskow and Rose [10] for a discussion of the differences.

16. Joskow and Rose [10] provide a discussion of this phenomenon.

17. See also Perl [11]. The only difference is that we express our costs in 1965 dollars rather than 1980 dollars using regional Handy-Whitman construction cost indices. The WAGE variable was calculated from Union Wages and Hours: Building Trades, July 1, 1965, U.S. Bureau of Labor Statistics, Bulletin 1487, Table 11, line 1.

18. The relevant thermodynamic considerations are discussed in Joskow and Rose [10].

19. We also experimented with specifications in which EFF and REL interacted with SIZE, but we encountered serious numerical problems in the computation of ML estimates. The textbook prescription for multicollinearity, which this problem resembles, is to gather more data. We are currently following this difficult prescription.

20. These patterns are consistent with the results of Joskow and Rose [10]. They have data for a larger sample of units coming on line from 1960 through 1980, they allow for learning-by-doing and environmental regulations, but they do not employ data on operating performance to control for quality differences. Joskow and Rose use individual-year dummy variables rather than the quadratic in TIME used here, but they nonetheless obtain estimated time effects qualitatively identical (during the 1960s) to those reported below.

21. The last three lines of Table II give coefficients of the indicated variables at the point of sample means. Note that the sample means of REL and EFF depend in general on the estimation method employed; the components of these vectors are treated as parameters to be estimated when ML is used.

22. Joskow and Rose [10] find that this pattern of increasing costs continues into the 1970s.

23. In the interactive specification, we had expected the coefficient of EFFxREL to be negative (building in high reliability should make efficiency more expensive on the margin), along with the coefficients of BTUxEFF (better coal should make efficiency less expensive on the margin) and BTUxREL. Only the last of these expectations was fulfilled.

24. Root-mean-squared percentage changes between fixed-effects (d*) and ML(d) estimates of EFF were 0.52 for the linear specification and 1.26 for the interactive specification. The corresponding statistics for REL were 0.04 and 5.49, respectively. The corresponding pairwise correlations all exceeded .988. (In contrast, the correlation between fixed-effects estimates of EFF and the corresponding unit means was .912, and the corresponding correlation for REL was only .849.)

25. Examples include Caves and Pugel [2], Comanor and Wilson [3], and Daskin [7].

Table I

First-Stage OLS Estimates of γ[a]

Independent Variables	Dependent Variable: REL	REL	EFF	EFF
∂CAPU	.0172 (.0036)	.0175 (.0036)	.0016 (.0012)	.0021 (.0013)
AGE	-.0152 (.0173)	-.0219 (.0182)	-.0073 (.0053)	-.0091 (.0056)
AGE^2	-.0039 (.0014)	-.0040 (.0014)	-.0010 (.0004)	-.0008 (.0004)
∂BTU		-3.7×10^{-5} (1.5×10^{-4})		-1.0×10^{-4} (4.7×10^{-5})
∂MOIST		.0461 (.0339)		-.0059 (.0103)
∂ASH		-.0373 (.0280)		-.0058 (.0086)
∂SULPH		.0866 (.0731)		.0232 (.0236)
$\sigma(\nu)$	.5143	.5097	.1478	.1469
R^2	.5438	.5538	.5639	.5735
T	530	530	438	438

[a]Standard errors are in parentheses; T is number of observations.

Table II

Second-Stage Estimates of Construction Cost Function Parameters (3)[a]

Variable		Linear Specification OLS-Means	OLS-d*	ML	Interactive Specification OLS-Means	OLS-d*	ML
CONST		10.68 (2.66)	11.05 (2.71)	11.03 (2.70)	22.68 (12.3)	21.51 (12.2)	21.40 (13.0)
SIZE		-.1626 (.0883)	-.1650 (.0927)	-.1639 (.0926)	-.1867 (.0903)	-.1810 (.0921)	-.1680 (.0889)
BTU		-.6127 (.266)	-.6221 (.264)	-.6205 (.264)	-1.876 (1.29)	-1.778 (1.29)	-1.812 (1.38)
WAGE		.6456 (.259)	.6494 (.257)	.6543 (.257)	.5473 (.269)	.5307 (.259)	.5055 (.263)
TIME		-.1521 (.0375)	-.1565 (.0380)	-.1568 (.0380)	-.1540 (.0378)	-.1662 (.0382)	-.1692 (.0379)
$TIME^2$		.0113 (.0029)	.0114 (.0029)	.0114 (.0029)	.0117 (.0029)	.0125 (.0029)	.0129 (.0029)
EFF		-.0529 (.223)	-.1330 (.191)	-.1568 (.191)	.5312 (21.5)	9.577 (17.6)	17.59 (18.6)
REL		-.0156 (.0824)	-.0058 (.0698)	-.0023 (.0697)	-8.465 (6.69)	-8.189 (4.95)	-10.77 (5.30)
EFFxREL					-.3474 (.365)	-.4314 (.279)	-.7303 (.301)
BTUxEFF					-.0149 (2.29)	-.9491 (1.87)	-1.752 (1.98)
BTUxREL					.9285 (.722)	.9100 (.533)	1.214 (.572)
$\sigma(\varepsilon)$		.1833	.1825	.1821	.1843	.1792	.1694
R^2		.4366	.4418	.4440	.4647	.4940	.5477
At point of means:[b]	BTU	-.6127	-.6221	-.6205	-.5731	-.7583	-.8025
	EFF	-.0529	-.1330	-.1568	-.0994	-.0682	-.0708
	REL	-.0156	-.0058	-.0023	-.0348	.0100	.0619

[a]Standard errors are in parentheses. ML standard errors and $\sigma(\varepsilon)$ adjusted for degrees of freedom to permit comparability with OLS estimates. Dependent variable is AVCOST.

[b]Coefficient of indicated variable at the point of sample means.